B.Sc.-M.Sc. Integrated Program
Semester-I Examination, 2021
Paper Title: Basics of Mathematics
Paper Code: BCH-GE-01-B (Th)

Time: 90 min

Max Marks: 37

Attempt all questions. Use only A4 sheets. All parts of a question must be attempted in continuation. Start answer of a fresh question on a new page. Cite examples and draw labelled diagrams wherever necessary.

Q1. Attempt All of the following: **(7 x 1 = 7)**

i) A square matrix A is called diagonal if .....

ii) If $A = \begin{bmatrix} a^{2021} & b^b \\ c & e^{2019} \end{bmatrix}$, then $(A^T)^T$ =....., where A^T is the transpose of A.

iii) The 2nd derivative of the function $y = x^{2021}$ is.....

iv) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$.

v) If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\, dx$ =.....

vi) If $y = e^{\sin^{-1} x}$, then $\frac{dy}{dx}$.....

vii) A function $F(x, y)$ is said to be homogeneous function of degree n if $F(tx, ty) =$ .....for any nonzero constant t.

Q2. Attempt ANY THREE of the following: **(3 x 4 = 12)**

i) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

ii) If x, y, z are different and if the determinant of the matrix $A = 0$, where $A = \begin{bmatrix} x & x^2 & 1 + x^2 \\ y & y^2 & 1 + y^2 \\ z & z^2 & 1 + z^2 \end{bmatrix}$, then prove that $1 + xyz = 0$.

iii) Evaluate $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$.

iv) Verify that the function $y = A\cos x + B\sin x$, where $a, b \in R$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$.

v) Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}$, provided that $y \neq 2$.

Q3. Attempt ANY THREE of the following: **(3 x 6 = 18)**

i) Show the differential equation $(x - y)dy - (x + 2y)dx = 0$ is homogeneous and solve it.

ii) Evaluate $\int_{-1}^{2} |x^3 - x| dx$.

iii) Find the inverse of the matrix A=$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$.

iv) Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & x \leq 2 \\ ax + b, & 2 < x \leq 10 \\ 21, & x \geq 10 \end{cases}$$

is continuous function.

v) If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\sin^{-1} t}}$, then find $\frac{dy}{dx}$.

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